Determining a Budget Profile from a Development Cost Estimate

David A. Lee
Program Analysis and Evaluation (Resource Analysis)
The Pentagon, Room 2D278
Washington, DC 20301-1800

Michael R. Hogue
Air Force Institute of Technology/Civilian Institutions Program
Wright-Patterson AFB, OH 45433-6583

Mark A. Gallagher
Program Analysis and Evaluation (Resource Analysis)
The Pentagon, Room 2D278
Washington, DC 20301-1800

Abstract

Often in developing budgets for programs with a development phase, analysts must determine the budget profile from a point estimate of the total development cost. Expenditures for Department of Defense (DoD) development programs, as recorded in Cost Performance Reports, are seen to fit a cumulative Rayleigh distribution reasonably well. Thus, given a point estimate of total development costs, a realistic expenditure profile can be determined using a Rayleigh model. Furthermore, these expenditures can be related to annual budget requirements through the DoD Comptroller's outlay rates. This paper describes a method for determining a budget profile from a point estimate of the total development cost.

1. Introduction

An approach to estimate the Total Obligation Authority (TOA) by fiscal years for an Engineering and Manufacturing Development (EMD) program is to estimate the total EMD cost and then to "spread" these costs over the projected years of the program. This paper outlines a procedure to spread a point estimate for total EMD funding to a budget profile. The procedure begins by modeling the cumulative EMD expenditures (also called outlays) in constant-year dollars with an appropriate Rayleigh cumulative probability distribution. The

expenditures are translated to TOA funds by solving a linear system of equations that relate outlay rates, TOA, and expenditures. The three major sections in this paper discuss the Rayleigh distribution as applied to EMD expenditures, estimating an expenditure stream, and determining a TOA profile. An example is used through the paper to demonstrate the technique.

2. Rayleigh Probability Distribution and EMD Expenditures

Norden [6] proposed that the Rayleigh probability density function could be used as a model of the manpower utilizations during a project. Putnam [7] applied the Rayleigh model to development of software projects. Watkins [8] and Abernethy [1] applied the model to defense acquisition data, and concluded that it fit their data well. Harmon, Ward, and Palmer [3] tested the ability of the Rayleigh model to estimate the costs of tactical weapon systems. Lee, Hogue, and Hoffman [5] discuss the agreement of the Rayleigh model with observed outlays in a wide variety of defense acquisition programs, and explain and offer resolution of some problems of extrapolation from data limited to early time values that were pointed out by Abernethy in Reference [1]. In this section, the Rayleigh distribution and its applicability for modeling EMD outlays are discussed.

Cumulative EMD expenditures are modeled with the cumulative Rayleigh distribution, which is expressed as follows:

$$F(t) = 1 - \exp(-at^2)$$

Figure 1. shows the variation of the cumulative Rayleigh distribution with the time-scale parameter a. As the value of a increases, the cumulative distribution rises faster and has a

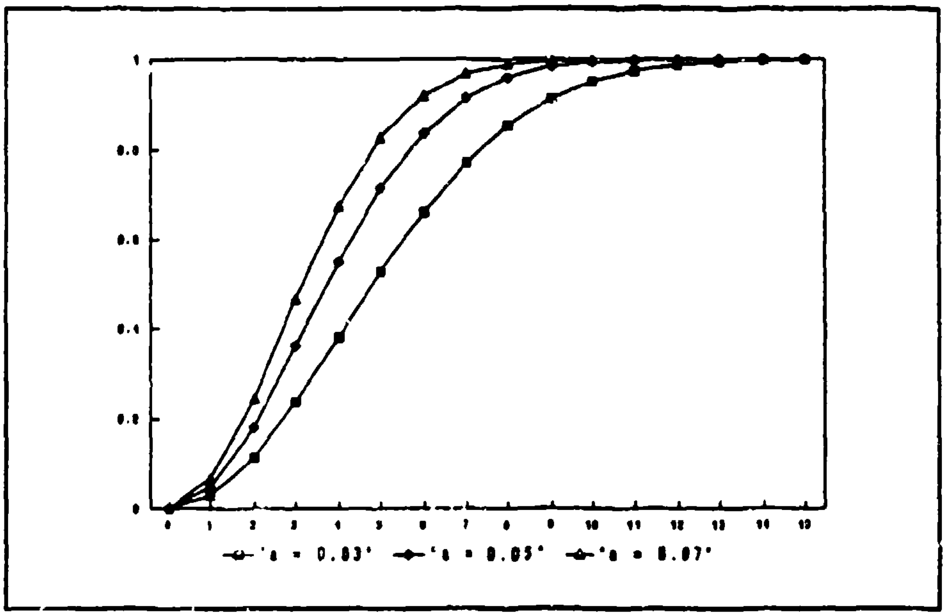

Figure 1. Cumulative Rayleigh Distributions

smaller tail. Probability densities are the derivative of the cumulative distributions. The Rayleigh probability density is

$$f(t) = 2\alpha t \exp(-\alpha t^2) \tag{1}$$

Figure 2 presents the Rayleigh probability densities corresponding to the cumulative

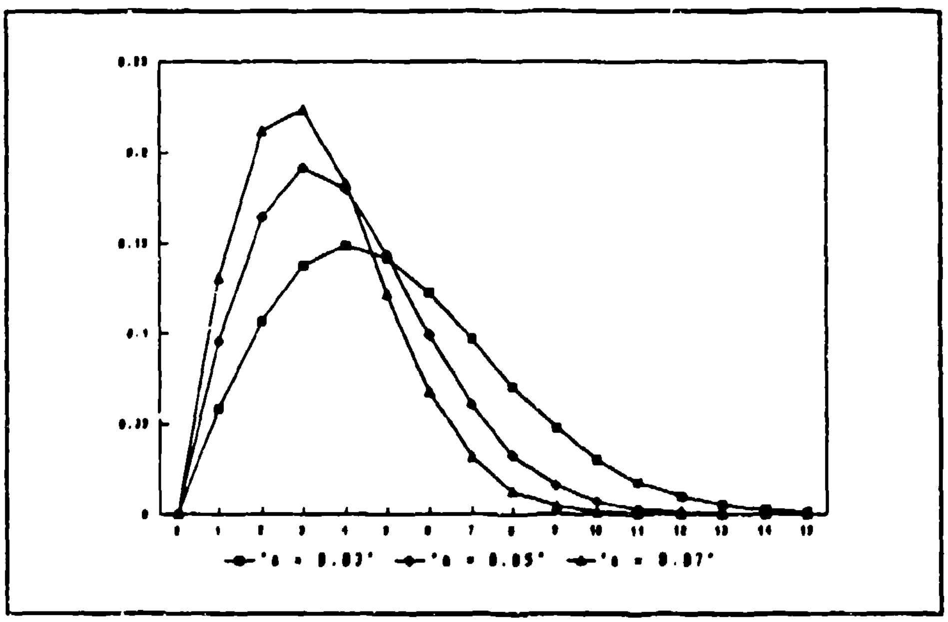

Figure 2. Rayleigh Probability Densities

distributions shown in Figure 1. As the parameter a increases, the peak of the distribution is early and higher. An additional parameter, d, is used to scale the Rayleigh distribution to the estimated total EMD cost. Thus, the cumulative expenditures, $E^*(t)$, are modeled with

$$E^*(t) = d(1-\exp(-at^2)) \tag{2}$$

The superscript * denotes base-year (equivalently, constant) dollar variables throughout this paper.

Figure 3, which is reproduced from Reference [5], shows how well the actual outlays from some 20 defense acquisition programs follow the Rayleigh model. We take these

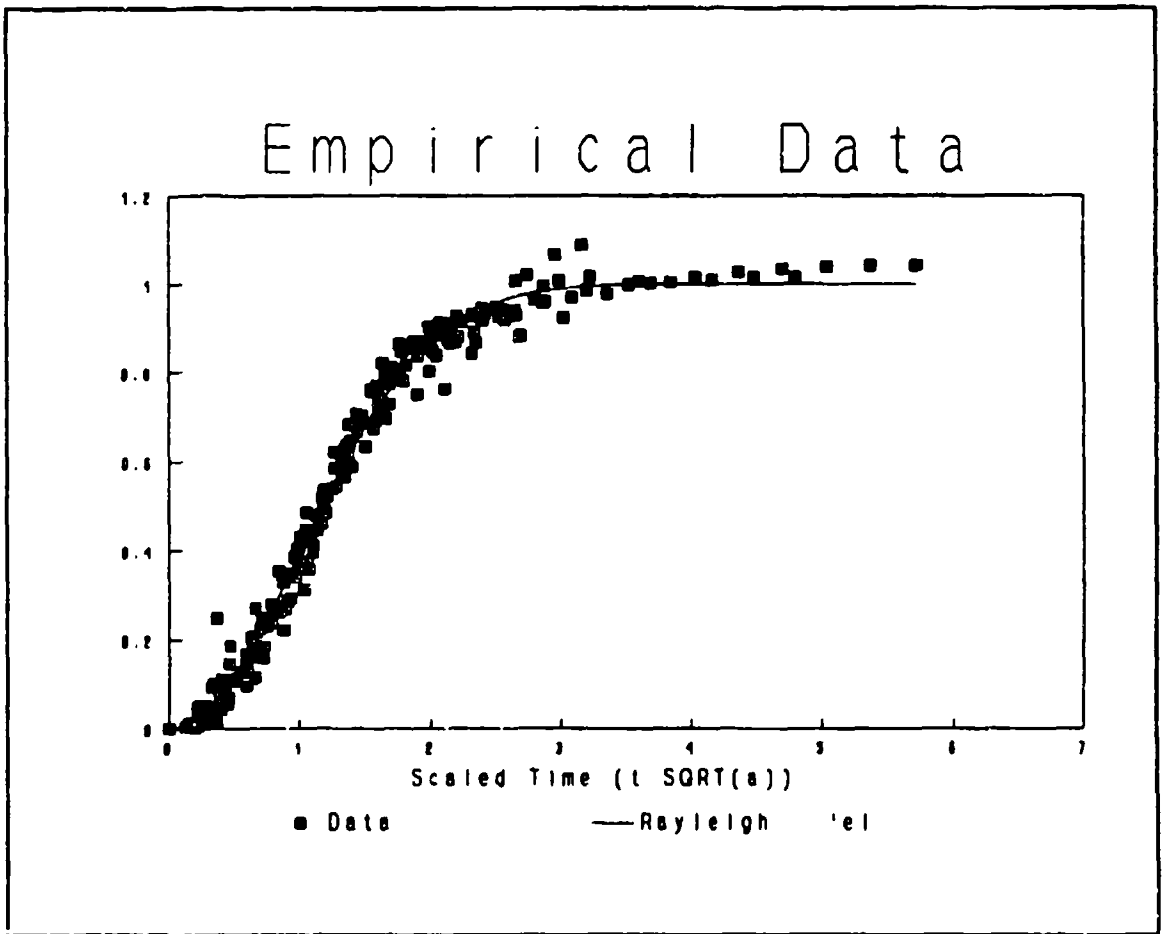

Figure 3. Empirical Data and Rayleigh Model

results, as well as those of references [2], [3], [4], and [8], as reasonably firm justification for using the Rayleigh model.

3. Outlay Determination

In order to apply this model to spread an estimate of total expenditures, the parameters a and d must be specified. The time scale parameter a of the Rayleigh distribution may be determined in several ways. One way is to specify the time of peak expenditure rate; this may be done with some confidence for certain classes of programs. For example, it is often observed that the peak expenditure rate in aircraft development programs comes at, or slightly before, the time of first flight. The Rayleigh parameter a is determined from the specified time of the peak expenditure rate by setting the derivative of the Rayleigh probability density, given in (2), equal to zero. This lead to

$$a = (1/2)(t_{peak})^{-2}$$

The time when the EMD effort is scheduled to complete, t_{fin}, can also be used to determine an appropriate value of a. Assume the time to the end of EMD is about three times the time to peak, $t_{fin} = 3t_{peak}$, and then

$$a = 4.5\ (t_{fin})^{-2}$$

The parameter d is generally selected to be approximately equal to the estimated total EMD expenditures. A value of d may be slightly greater than the total EMD expenditures estimated to reduce the effect of the Rayleigh distribution's infinite tail. In addition, if the EMD program does not begin at the start of the fiscal year, the appropriate value for t for the first year may be a fraction. As an example, suppose a program begins in March, t should

equal 0.5 for the first fiscal year. The value of t for subsequent fiscal years should be incremented by 1.0, such as 1.5, 2.5, 3.5, and so on.

It is also possible to determine a and d together, with the rule of thumb known to many experienced cost analysts, that roughly 60% of a development program's total expenditures are made in the first half of the development period. The requirement that this is so, and that a total amount D is expended at time t_{fin}, leads to the two equations

$$d\,(1 - e^{-\frac{at_f^2}{4}}) = 0.6D$$

and

$$d\,(1 - e^{-at_f^2}) = D$$

These two equations may be solved numerically, to give

$$a = \frac{3.4854}{t_f^2}$$

and

$$d = 1.0316\ D$$

Assume that an example EMD program is estimated to cost a billion dollars (expressed in terms of thousands of dollars in this paper) with the funding for EMD occurring in 8.5 years. The funding is in fiscal years 1995 through 2003. In this example, we determined the parameters based on the time of completion and the projection for total EMD funding. Since t_{fin} is 8.5, a equals 0.062. Since the outlay pattern indicates that the TOA for each year is obligated over a 5 year period, 9 years of EMD funding results in 13 years of expenditures.

With a = 0.062 and d = 1000, the Rayleigh model in (2) results in the following expenditures profile shown in Table 1.

The estimated annual expenditure in base-year dollars, defined as O^*_i for each year, is determined by taking the difference between the current and previous cumulative expenditures.

$$O_i^* = E^*(i) - E^*(i-1)$$

Since no adjustment has been made for inflation, these annual expenditures are in constant dollars in the same units as d. Furthermore, since d is determined from the EMD point

Table 1. Example Outlay Profile

Year	Time	Cumulative Raleigh	Annual Outlay
1995	0.5	15.74	15.74
1996	1.5	133.05	117.31
1997	2.5	327.39	194.34
1998	3.5	540.36	212.97
1999	4.5	723.34	182.97
2000	5.5	853.32	129.98
2001	6.5	931.50	78.18
2002	7.5	971.83	40.32
2003	8.5	989.79	17.97
2004	9.5	996.74	6.95
2005	10.5	999.08	2.34
2006	11.5	999.77	0.69
2007	12.5	999.95	0.18

estimate, these annual outlays are expressed in the base-year dollars of that point estimate. These outlays could be inflated to current dollars prior to determining TOA or the TOA can be estimated in constant dollars and then inflated. The equivalence of these approaches is demonstrated in the Appendix.

4. Total Obligation Authority (TOA) Determination

At this point, we have generated a constant-dollar outlay stream corresponding to the Rayleigh model. Often, cost analysts require not outlays, but TOA. The OSD comptroller publishes tables of standard outlay patterns for various accounts. These outlay patterns permit the translations of TOA to outlays. For example, the comptroller's standard for Navy RDT&E outlay pattern to three significant digits is (0.544, 0.339, 0.079, 0.013, 0.025). That is, \$1,000 (base-year dollars) in TOA for a given year generates outlays of \$544 in that year, \$339 in expenditures in the next year, and so on through \$25 in the fifth year. Inflation plays a role, of course, and so, if c_i denotes the appropriate cumulative inflation index from the base year to year i, the \$1,000 in year k generates the outlay stream of (\$544c_k, \$339c_{k+1}, \$79c_{k+2}, \$13c_{k+3}, \$25c_{k+4}).

However, our problem is to "go the other way," and, given our outlay stream, generate the corresponding TOA stream. Let T_i^* represent the TOA and O^*_i represent the outlay in year i, expressed in base-year dollars. Furthermore, let $(s^*_1, s^*_2, ..., s^*_J)$ represent the outlay pattern occurring over J years. Since TOA funding results in outlays in that year, TOA and outlays begin in the same fiscal year. Thus, the first TOA can be determined by taking the desired outlay and dividing by the first term in the outlay pattern; $T_1^* = O^*_1/s^*_1$. For the

example with the Navy RDT&E outlay pattern, $T_1^* = 15.74/0.544 = 28.93$. The outlay for the second year must be reduced for the outlay which occurs from the TOA in the first year; $T_2^* = (O^*_2 - s^*_2 T_1^*)/s^*_1$. For the example problem, $T_2^* = (117.31 - 0.339(28.93))/0.544 = 197.62$. In general, each subsequent year needs to account for the outlays from TOA in prior years with:

$$T_k^* = (O^*_k - s^*_2 T_{k-1}^* - s^*_3 T_{k-2}^* - \ldots - s^*_J T_{k-J+1}^*)/s^*_1$$

where T_i^* for $i <= 0$ is zero. The desired TOA funding profile is determined by iteratively solving for each year's TOA.

Since each year is represented by a linear equation, one approach to solving the problem is to set up a system of equations. The dimension of the system is the number of years of expenditures. In the example outlays shown in Table 1, the dimension is 13. Let **A** denote a matrix based on the outlay rates. The *i*th column of **A** consists of zeros for the rows less than *i*, the appropriate outlay rates beginning in the *i*th row, and zeros for any rows after the last outlay rate. For the example problem with the Navy RDT&E outlay pattern, the resulting matrix **A** is:

0.544	0	0	0	0	0	0	0	0	0	0	0	0
0.339	0.544	0	0	0	0	0	0	0	0	0	0	0
0.079	0.339	0.544	0	0	0	0	0	0	0	0	0	0
0.013	0.079	0.339	0.544	0	0	0	0	0	0	0	0	0
0.025	0.013	0.079	0.339	0.544	0	0	0	0	0	0	0	0
0	0.025	0.013	0.079	0.339	0.544	0	0	0	0	0	0	0
0	0	0.025	0.013	0.079	0.339	0.544	0	0	0	0	0	0
0	0	0	0.025	0.013	0.079	0.339	0.544	0	0	0	0	0
0	0	0	0	0.025	0.013	0.079	0.339	0.544	0	0	0	0
0	0	0	0	0	0.025	0.013	0.079	0.339	0.544	0	0	0
0	0	0	0	0	0	0.025	0.013	0.079	0.339	0.544	0	0
0	0	0	0	0	0	0	0.025	0.013	0.079	0.339	0.544	0
0	0	0	0	0	0	0	0	0.025	0.013	0.079	0.339	0.544

The ith row of **A** is the fractions of TOA that contribute to the outlay in the ith year, O^*_i. The ith column of **A** represents how the TOA in the ith year, T^*_i, is distributed to outlays in various years. The matrix **A** is a lower triangular matrix since the upper right elements are all zeros.

Let $\mathbf{O}^*$ be the vector of outlays in base-year dollars, and $\mathbf{T}^*$ be the vector of TOA funding in base-year dollars. Outlays can be converted to TOA by setting up and solving a system of linear equations: $\mathbf{AT}^* = \mathbf{O}^*$. While the funding estimates could be solved by inverting the matrix **A**, since **A** is a triangular matrix the system can be solved iteratively, as shown previously. For the example, the desired outlay profile is shown in Table 1, the TOA profile and the expected outlays are shown in Table 2. The expected outlays are determined by applying the outlay rates to the calculated TOA values, and the error is the difference between the desired outlays and the TOA.

Table 2. Example of TOA Profile

Year	Desired Outlay	TOA	Expected Outlay	Error
1995	15.74	28.93	15.74	0.000
1996	117.31	197.62	117.31	0.000
1997	194.34	229.90	194.34	0.000
1998	212.97	218.84	212.97	0.000
1999	182.97	160.5[illegible]	182.97	0.000
2000	129.98	92.54	129.98	0.000
2001	78.18	46.94	78.18	0.000
2002	40.32	17.54	40.32	0.000
2003	17.97	5.69	17.97	0.000
2004	6.95	1.31	6.95	0.000
2005	2.34	0.09	2.34	0.000
2006	0.69	0.08	0.69	0.000
2007	0.18	0.00	0.18	0.000
2008	0.00	0.00	0.03	0.000
2009	0.00	0.00	0.00	0.000
2010	0.00	0.00	0.00	0.000
2011	0.00	0.00	0.00	0.001
Total	999.95	999.98	999.98	0.001

The EMD program used in this example specified a funding duration of nine years (1995-2003), but the resulting program has twelve years of funding. The additional years result from using the entire system of equations. If the TOA is set to zero after the desire funding program, the profile in Table 3 is achieved.

Table 3. Example TOA Stream (Truncated After 9 Years)

Year	Outlay	TOA	Outlays	Error
1995	15.74	28.93	15.74	0.000
1996	117.31	197.62	117.31	0.000
1997	194.34	229.90	194.34	0.000
1998	212.97	218.84	212.97	0.000
1999	182.97	160.54	182.97	0.000
2000	129.98	92.54	129.98	0.000
2001	78.18	46.94	78.18	0.000
2002	40.32	17.54	40.32	0.000
2003	17.97	5.69	17.97	0.000
2004	6.95	0.00	6.24	0.711
2005	2.34	0.00	1.85	0.491
2006	0.69	0.00	0.51	0.176
2007	0.18	0.00	0.14	0.035
2008	0.04	0.00	0.00	0.040
2009	0.01	0.00	0.00	0.008
2010	0.00	0.00	0.00	0.001
2011	0.00	0.00	0.00	0.000
Total	1000.00	998.54	998.54	1.463

The resulting error of truncating the TOA sequence is less for models with larger values for the parameter a since the tail of the outlay distribution is smaller. The truncation induces a small error between the desired outlays and expected outlays, but this error is relatively small.

These annual expenditures, also called outlays, and the TOA in constant dollars are depicted in Figure 4.

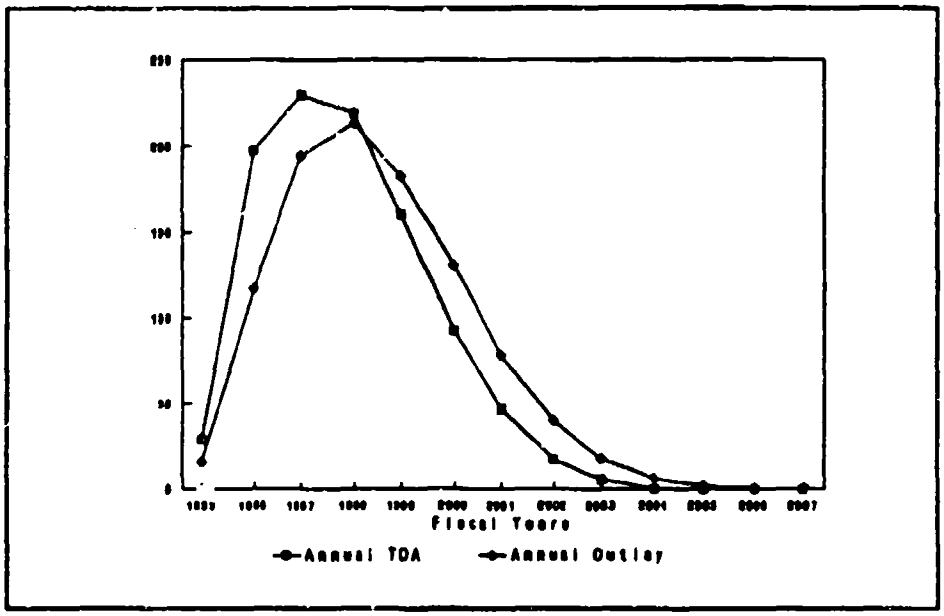

Figure 4. Annual TOA and Outlays (Constant Dollars)

In this example, the estimated expenditures and funding estimates are in expressed in the base year dollars of the EMD estimate. This procedure has spread the EMD estimate into a budget profile but has not adjusted for inflation. If a then-year profile is desired, each of the TOA estimates needs to be adjusted from the base-year dollars of the original EMD total cost estimate to the funding year with the appropriate inflation indices. Table 4 shows the TOA and outlays in then-year dollars (under the assumption that the base-year was FY91). The TOA was inflated with weighted indices, which account for the outlay pattern, while the outlays were inflated with raw indices. The assumed inflation was 3.21 percent for the years 1994 and later.

Table 4. TOA and Outlays (Current Dollars)

Year	TOA	Outlays
1995	32.91	17.90
1996	232.00	137.69
1997	278.57	235.41
1998	273.66	266.26
1999	207.19	236.09
2000	123.27	173.10
2001	64.53	107.45
2002	24.88	57.20
2003	8.34	26.30
2004	0.00	9.43
2005	0.00	2.89
2006	0.00	0.82
2007	0.00	0.24

Figure 5 is a graph of the funding profile and outlays in current dollars.

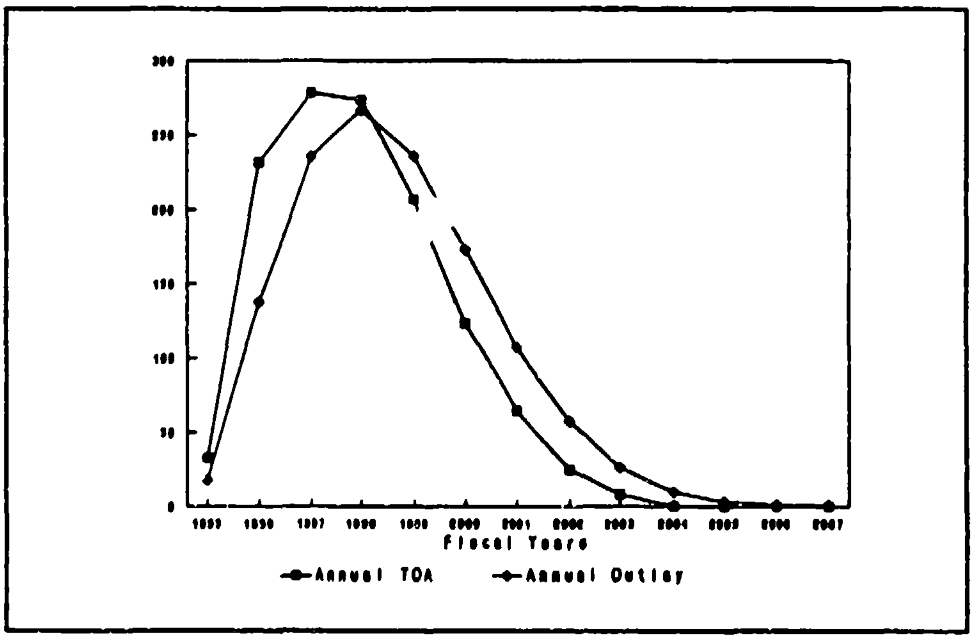

Figure 5. Annual TOA and Outlays (Current Dollars)

5. Summary

An approach to spread an EMD point estimate into a budget profile has been described. The procedure entails modeling the expenditures with a Rayleigh probability distribution. After the annual outlays are determined, a system of linear equations relating outlay rates, TOA, and expenditures is solved to determine annual funding. If current dollar values are desired, the TOA and outlays can be inflated from constant dollars either before setting up the system of equations or after solving the system. This procedure has the advantage that the Rayleigh distribution, which fits empirical data very well, is employed and that the length or peak of the EMD program can be specified.

Appendix

This appendix shows that the procedure in this note, of determining TOA in base year dollars and then inflating to then-year dollars, is equivalent to a more traditional view of TOA. In that view, TOA is in then-year dollars, and is spent, in accordance with the assigned spend-out pattern, to generate an outlay stream.

Let us denote by T_i the TOA in year i, in dollars of year i. Let O_i^* denote the base-year outlay required in year i, and let O_i denote the equivalent then-year outlay for year i. Let c_i denote cumulative inflation from the base year to year i. Let s_j^* denote the fraction of TOA value to be spent in year j. Then the statement that outlays match requirements is

$$\sum_{j=0}^{J} T_{i-j}\, s_j^* \frac{c_i}{c_{i-j}} = O_i^*\, c_i = O_i \qquad (3)$$

where J is the number of years in the spend-out pattern. (The fraction of T_i spent in year j could be represented as $s_{i,j} = s_j^* \frac{c_i}{c_{i-j}}$, but this outlay pattern may be different for each year because of the inflation indices.) Canceling the common factor c_i from both sides of the above equation and grouping terms in the sum gives

$$\sum_{j=0}^{J} \frac{T_{i-j}}{c_{i-j}}\, s_j^* = \sum_{j=0}^{J} T_{i-j}^* s_j^* = O_i^* \qquad (4)$$

where T_i^* is TOA expressed in base-year dollars. Thus, TOA (in then-year dollars) can be related to outlays in then-year dollars, as in (3), or TOA converted to constant dollars can be related to outlays in base-year dollars as shown in (4).

References

[1] Abernethy, T., *An Application of the Rayleigh Distribution to Contract Cost Data*, Master's Thesis, Naval Postgraduate School, Monterey, California, 1984

[2] Elrod, Suzanne M., *Engineering and Manufacturing Development Cost Estimation: An Analysis of Combined Time-Phased and Classical Techniques*, Master's Thesis, Naval Postgraduate School, Monterey, California, June, 1992.

[3] Harmon, B. R., Ward, L. M., Palmer, P. R., "Schedule Estimating Relationships for Tactical Aircraft," T. R. Gulledge, L. A. Litteral (Editors), *Lecture Notes in Economics and Mathematical Systems*, pp. 259-280, Springer-Verlag, 1989.

[4] Hogue, Michael R., "Rayleigh Curve Fitting to R&D Cost Profiles", unpublished report, OASD/PA&E, The Pentagon, June, 1990.

[5] Lee, D., Hogue, M., and Hoffman, D., *Time Histories of Expenditures for Defense Acquisition Programs in the Development Phase*, presented at the 1993 Annual Meeting of the International Society for Parametric Analysis

[6] Norden, Peter V., "Useful Tools for Project Management," in *Management of Production*, M. K. Starr, Editor., Baltimore, Maryland, Penguin, 1970, pp.71-101.

[7] Putnam, Lawrence H., "A General Empirical Solution to the Macro Software Sizing and Estimating Problem," *IEEE Transactions on Software Engineering*, July 1978, pp. 345-361.

[8] Watkins, Harry, *An Application of Rayleigh Curve Theory to Contract Cost Estimation and Control*, Master's Thesis, Naval Postgraduate School, Monterey, California, March, 1982.

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY *(Leave blank)*

2. REPORT DATE
September 1993

3. REPORT TYPE AND DATES COVERED
ADODCAS 1993

4. TITLE AND SUBTITLE
Determining a Budget Profile from a Development Cost Estimate

5. FUNDING NUMBERS

6. AUTHOR(S)
David Lee
Michael Hogue
Mark Gallagher

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Program Analysis & Evaluation
The Pentagon, Room 2D278
Washington, DC 20301-1800

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES
Annual Department of Defense Cost Analysis Symposium Paper

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Statement A: Approved for Public Release; Distribution is Unlimited

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*
See Over

14. SUBJECT TERMS
Cost Estimate
Budget

15. NUMBER OF PAGES

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
U	U	U	UL

NSN 7540-01-280-5500

Standard Form 298 (Rev. 2-89)
Prescribed by ANSI Std. Z39-18
298-102

DETERMINING A BUDGET PROFILE FROM A DEVELOPMENT COST ESTIMATE

Often in developing budgets for programs with a development phase, analysts must determine the budget profile from a point estimate of the total development cost. Expenditures for Department of Defense (DoD) development programs, as recorded in Cost Performance Reports, are seen to fit a cumulative Rayleigh distribution reasonably well. Thus, given a point estimate of total development costs, a realistic expenditure profile can be determined using a Rayleigh model. Furthermore, these expenditures can be related to annual budget requirements through the DoD Comptroller's outlay rates. This paper describes a method for determining a budget profile from a point estimate of the total development cost.

David A. Lee
Cost Analysis Improvement Group
The Pentagon, Rm 2D278
Washington, DC 20301-1800

Michael R. Hogue
Air Force Institute of Technology/Civilian Institute Program
Wright-Patterson AFB, OH 45433-6583

Capt. Mark A. Gallagher
Weapons Systems Cost Analysis Division
Office Director Program Analysis and Eval
1800 Defense Pentagon
Washington DC 20301-1800